Correction to 1062. *Proposed by D. M. Bătineţu-Giurgiu, Matei Basarab National College, Bucharest, Romania, and Neculai Stanciu, George Emil Palade Secondary School, Buzău, Romania.*

Consider a nonisosceles triangle with sidelengths a, b, c and area S. Prove that

(a) $$\frac{a^6}{(a^2-b^2)(a^2-c^2)}+\frac{b^6}{(b^2-a^2)(b^2-c^2)}+\frac{c^6}{(c^2-a^2)(c^2-b^2)}>4\sqrt{3}S.$$

(b) $$\frac{a^6}{(a-b)^2(a-c)^2}+\frac{b^6}{(b-a)^2(b-c)^2}+\frac{c^6}{(c-a)^2(c-b)^2}>4\sqrt{3}S.$$

This correction to Problem 1062 appeared in the March 2016 issue of *The College Mathematics Journal*. Its solution will be published in the March 2017 issue.

An inequality for nonisosceles triangles

1063. *Proposed by D. M. Bătinetu-Giurgiu, Matei Basarab National College, Bucharest, Romania, and Neculai Stanciu, George Emil Palade Secondary School, Romania.*

Let ABC be a nonisosceles triangle with sides a, b, c and inradius r. Prove that

$$\frac{a^8}{(b+c)(a-b)^2(a-c)^2}+\frac{b^8}{(a+c)(b-a)^2(b-c)^2}+\frac{c^8}{(a+b)(c-a)^2(c-b)^2}>144\sqrt{3}r^3.$$

Solution by Arkday Alt, San Jose, CA.

By the Cauchy–Schwarz inequality,

$$\sum_{\text{cyc}}\frac{a^8}{(b+c)(a-b)^2(a-c)^2}\geq\frac{1}{\sum_{\text{cyc}}(b+c)}\cdot\left(\sum_{\text{cyc}}\frac{a^4}{(a-b)(a-c)}\right)^2$$

or equivalently

$$\sum_{\text{cyc}}\frac{a^8}{(b+c)(a-b)^2(a-c)^2}\geq\frac{1}{4s}\left(\sum_{\text{cyc}}\frac{a^4}{(a-b)(a-c)}\right)^2.$$

where s is the semiperimeter. Since

$$\sum_{\text{cyc}}\frac{a^4}{(a-b)(a-c)}=a^2+b^2+c^2+ab+ac+bc,$$

$$\sum_{\text{cyc}}\frac{a^8}{(b+c)(a-b)^2(a-c)^2}\geq\frac{(a^2+b^2+c^2+ab+ac+bc)^2}{4s}.$$

Note that

$$a^2+b^2+c^2+ab+ac+bc\geq\frac{2}{3}(a+b+c)^2=\frac{8s^2}{3}.$$

Indeed,

$$3(a^2+b^2+c^2+ab+ac+bc)-2(a+b+c)^2 = a^2+b^2+c^2-ab-bc-ca$$

is nonnegative. The expression is zero if and only if $a=b=c$, but triangle ABC is nonisosceles. Therefore,

$$\sum_{\text{cyc}} \frac{a^8}{(b+c)(a-b)^2(a-c)^2} > \frac{\left(\frac{8s^2}{3}\right)^2}{4s} = \frac{16s^3}{9}.$$

Noting that $s \geq 3\sqrt{3}r$, we finally obtain

$$\sum_{\text{cyc}} \frac{a^8}{(b+c)(a-b)^2(a-c)^2} > \frac{16s^3}{9} \geq \frac{16(3\sqrt{3}r)^3}{9} = 144\sqrt{3}r^3.$$

Also solved by the proposer. One incorrect solution was received.

Correction to 1064. *Proposed by Mircea Merca, University of Craiova, Romania.*

Let n be a positive integer. Prove that

$$0 < \frac{1}{\pi} \cdot \frac{2^{2n}}{\binom{2n}{n}} - \sum_{k=1}^{n} \cos^{2n+1}\left(\frac{k\pi}{2n+1}\right) < 1.$$

This correction to Problem 1064 appeared in the March 2016 issue of *The College Mathematics Journal*. Its solution will be published in the March 2017 issue.

Two inequalities for complex numbers

1065. *Proposed by José Díaz-Barrero, Technical University of Catalonia, Barcelona, Spain.*

Let $n \geq 0$ be an integer, and let $\alpha, a_0, a_1, \ldots, a_n, b_0, b_1, \ldots, b_n$ be complex numbers. Prove that

(a) $\operatorname{Re}\left(\overline{\alpha}\sum_{k=0}^{n} a_k b_k\right) \leq \frac{1}{2}\left(\sum_{k=0}^{n}|a_k|^2 + |\alpha|^2\sum_{k=0}^{n}|b_k|^2\right).$

(b) $\operatorname{Re}\left(\sum_{k=0}^{n} a_k b_k\right) \leq \frac{1}{2(n+1)}\left(\sum_{k=0}^{n}|a_k|^2 + \frac{(2n+1)(2n+3)}{3}\sum_{k=0}^{n}|b_k|^2\right).$

Solution by Eugene Herman, Grinnell College, Grinnell, IA.

We prove stronger versions of both (a) and (b):

(a′) $\left|\overline{\alpha}\sum_{k=0}^{n} a_k b_k\right| \leq \frac{1}{2}\left(\sum_{k=0}^{n}|a_k|^2 + |\alpha|^2\sum_{k=0}^{n}|b_k|^2\right).$

(b′) $\left|\sum_{k=0}^{n} a_k b_k\right| \leq \frac{1}{2K}\left(\sum_{k=0}^{n}|a_k|^2 + L\sum_{k=0}^{n}|b_k|^2\right)$, whenever $0 < K$ and $K^2 \leq L$.